Proofs

Proove that for all integers n, if n is even then n^2 is even.

Suppose n is an integer, let n be even, $n = 2k$ for some integers k. Then $n^2 = (2k)n = 2(kn)$ where kn is an integer so n^2 is even.

{n is an intger
n is even

↓

$n = 2k$ k is an integer

↓

$n^2 = 2k \cdot n$

$n^2 = 2(kn)$ intger

Proof by Cases

True or false?

For all integers n, n^2+n is even.

This statement is true. Suppose n is an integers, we have 2 cases.

Case 1

n is even, then $n = 2k$ for some integer k and $n^2+n = n(n+1) = 2k(n+1)$, value $k(n+1)$ is an integer then n^2+n is even.

Case 2

n is odd, then $n = 2k+1$ for some integer k. Then $n^2+n = n(n+1) = n(2k+1+1) = 2n(k+1)$ where $n(k+1)$ is an integer, so n^2+n is even.

True or False

For all integers n, there exists an integers m so that $n+m$ is even.

This statement is true. Suppose n is an integers, $n+m=2$
Choose $m=2-n$, then m is an integers, $m=2-n$
$n+m = n+2-n = 2 = 2\times 1$ where 1 is an integer, so $n+m$ is even.

True or False

There exists an integer m so that for all integers n, $n+m$ is even.

This statement is false

Prove the Negation:

"For all integers m, there exists an integers n, such that $n+m$ is odd."

Same proof.

True or False?

For all integers y, there exists an integer x so that $x^2+x=y$.

This statement is false.

Negation:

There exis an integers y, so that for all integers x, $x^2+x \neq y$.

proof:

Choose $y=-1$, then y is an integer, then
$x^2+x = x^2+2(\frac{1}{2}x)+\frac{1}{4}-\frac{1}{4} = (x+\frac{1}{2})^2-\frac{1}{4} \geq -\frac{1}{4} > -1 = y$. Thus, $x^2+x\neq y$

A prime number is an integer larger than 1 so that its only positive divisors are 1 and itself.